

Influence of Hydrodynamic Fluctuations on the Phase Transition in Models E and F of Critical Dynamics.

M. Dančo¹, M. Hnatich^{1,2}, M. V. Komarova³, D. M. Krasnov³,
T. Lučivjanský^{1,2}, L. Mižišin², M. Yu. Nalimov³

¹ Institute of Experimental Physics, Slovak Academy of Sciences,
Watsonova 47, 040 01 Košice, Slovakia,

² Faculty of Sciences, P.J. Šafarik University,
Moyzesova 16, 040 01 Košice, Slovakia,

³ Department of Theoretical Physics, St. Petersburg University,
Ulyanovskaya 1, St. Petersburg, Petrodvorets, 198504 Russia.

August 26, 2016

Abstract

We use the renormalization group method to study model E of critical dynamics in the presence of velocity fluctuations arising in accordance with the stochastic Navier-Stokes equation. Using Martin-Siggia-Rose theorem, we obtain a field-theoretical model that allows a perturbative renormalization group analysis. By direct power counting and an analysis of ultraviolet divergences, we show that the model is multiplicatively renormalizable, and we use a two-parameter expansion in ϵ and δ to calculate renormalization constants. Here, ϵ is a deviation from the critical dimension four, and δ is a deviation from the Kolmogorov regime. We present the results of the one-loop approximation and part of the fixed-point structure. We briefly discuss the possible effect of velocity fluctuations on the large-scale behavior of the model.

1 Introduction

Bose condensation is an important physical phenomenon observed nowadays not only in the superfluidity of liquid helium but also in the condensation of inert gases [1].

According to [2], the critical dynamics near such a phase transition can be described using model F. This model was analyzed in [3] using the renormalization group (RG) approach. It was shown that in the critical region, model F is equivalent to model E (according to the standard terminology introduced in [2])

Both dynamical models E and F of critical dynamics are free from hydrodynamic modes because velocity field turns out to be infrared (IR) irrelevant in the critical range. Therefore, the critical exponent (e.g. for the viscosity) is still unknown, although the viscosity vanishes during the considered phase transition and manifests the features of order parameter. Moreover, the problem of the influence of turbulence on the phase transition into the superfluid state remains unsolved.

A stochastic equation for the critical dynamics of a Bose system in the presence of a random velocity field was proposed in [4]. Such a modification of model E leads to some deviations from the standard field-theoretical approach, and we adopt it. We here continue the investigation begun in [4]. Our aim is to study different scaling regimes of the proposed model.

This paper is structured as follows. We begin by analyzing the field theory formulation of the model and its renormalization (see Sec. 2). In Sec. 3, we present some interesting details of the one-loop calculation and give relations between renormalization constants. In Sec. 4, we analyze the fixed points and their regions of IR stability and give the results of the one-loop calculations of the RG functions. In Sec. 5, we present brief conclusions.

2 Field-theoretic formulation of the model

The stochastic equations of Bose-like systems can be described in the vicinity of their critical points [4] by the equation

$$\begin{aligned} \partial_t \psi + \partial_i (v_i \psi) = & \lambda(1 + ib)[\partial^2 \psi - g_1(\psi^+ \psi)\psi/3 + g_2 m \psi] \\ & + i\lambda g_3 \psi [g_2 \psi^+ \psi - m + h] + f_{\psi^+}, \end{aligned} \quad (1)$$

and by the analogous equation for the complex conjugate field ψ^+ . The fields ψ , ψ^+ represent order parameter (averages of field operators of Bose particles). The field m is a linear combination of internal energy and density [2] and related to fluctuations of temperature of the considered system; its evolution is described by

$$\partial_t m + \partial_i (v_i m) = -\lambda u \partial^2 [g_2 \psi^+ \psi - m + h] + i\lambda g_3 m + f_m. \quad (2)$$

The field v is the fluctuating velocity field (transverse due to incompressibility) and behaves according to

$$\begin{aligned} \partial_t v + \partial_i(v_i v) = & \nu \Delta v - \psi^+ \partial[\partial^2 \psi - \frac{g_1}{3}(\psi^+ \psi)\psi + g_2 m \psi] \\ & - \psi \partial[\partial^2 \psi^+ - \frac{g_1}{3}(\psi^+ \psi)\psi^+ + g_2 m \psi^+] - m \partial[g_2 \psi^+ \psi - m + h] + f_v. \end{aligned} \quad (3)$$

The random forces $f_i, i \in \{\psi^+, \psi, m, v\}$ are assumed to be Gaussian random variables with zero means and correlators D_i :

$$D_\psi(p, t, t') = \lambda \delta(t - t'), \quad D_m(p, t, t') = \lambda u p^2 \delta(t - t'), \quad D_v(p, t, t') = g_4 \nu^3 p^{\epsilon - \delta} \delta(t - t'). \quad (4)$$

To analyze the model, we use dimensional regularization (see below) around its critical dimension four with the standard ε -expansion (where ε is defined by $d = 4 - \varepsilon$). The parameter δ measures the deviation from the Kolmogorov regime, i.e. the value $\delta = -3$ (and $\varepsilon = 1$) corresponds to the inclusion of equilibrium fluctuations of velocity, and $\delta = 4$ defines the regime of developed turbulence [5, 6, 7]. We note that Eq.(3) is the stochastic Navier-Stokes equation with added terms ensuring the existence of an equilibrium statistical limit for the proposed model. An important physical fact is that only the noise D_v determines which specific hydrodynamic regime is realized.

Our considerations are based on a modification of model E, not only because it is relatively simple but also because it was shown in [3] that this model corresponds to the stable IR-scaling regime in model F [2]. The standard Martin-Siggia-Rose formalism (MSR) [8] for the system (1) leads to the field-theoretic action of the form

$$\begin{aligned} S = & 2\lambda \psi^{+'} \psi' - \lambda u m' \partial^2 m' + v' D_v v' + \psi^{+'} \{-\partial_t \psi - \partial_i(v_i \psi) + \\ & + \lambda[\partial^2 \psi - g_1(\psi^+ \psi)\psi/3] + i\lambda g_3 \psi[-m + h]\} + \\ & + \psi' \{-\partial_t \psi^+ - \partial_i(v_i \psi^+) + \lambda[\partial^2 \psi^+ - g_1(\psi^+ \psi)\psi^+/3] - \\ & - i\lambda g_3 \psi^+[-m + h]\} + m' \{-\partial_t m - \partial_i(v_i m) - \lambda u \partial^2[-m + h] + \\ & + i\lambda g_5[\psi^+ \partial^2 \psi - \psi \partial^2 \psi^+]\} + v' \{-\partial_t v + \nu \Delta v - \partial_i(v_i v)\}, \end{aligned} \quad (5)$$

where integrations over spacetime (t, \mathbf{x}) and summations over repeated vector indices are understood. The terms

$$v' \left\{ -\psi^+ \partial[\partial^2 \psi - \frac{g_1}{3}(\psi^+ \psi)\psi] - \psi \partial[\partial^2 \psi^+ - \frac{g_1}{3}(\psi^+ \psi)\psi^+] - m \partial[-m + h] \right\} \quad (6)$$

are not included in action (5), because it can be shown that they are IR-irrelevant.

The renormalization of the proposed model was described in detail in [4]. In the renormalization group analysis, the following properties of the model must be applied:

- Galilean invariance is present;
- nonlocal counterterms of the type $v'D_v v'$ are absent;
- the dimensionless constant ν is expressed in the form $\nu = u_1 \lambda$ with u_1 and is considered a new charge of the model with its own renormalization constant;
- counterterms of the type $v'\partial_t v$ and $v'(v\partial v)$, are absent, as is usual in developed turbulence;
- the derivative in interaction terms $\phi'\partial_i(v_i\phi)$ can always be transferred to the field ϕ' or ϕ using integration by parts.

In the studied model, the connection with statics is violated (because the form of the correlator D_v changes). Nevertheless, it was shown that the multiplicative renormalization can be recovered by adding one new charge at interaction $g_5 m'(\psi^+ \partial^2 \psi - \psi \partial^2 \psi^+)$, i.e., its bare action is related to the renormalized action by the usual multiplicative relations for the fields and parameters:

$$\begin{aligned}
S_R(\varphi) &= S(Z_\varphi \varphi), \\
Z_\varphi \varphi &\equiv \left\{ Z_\psi \psi, Z_{\psi'} \psi', Z_{\psi^+} \psi^+, Z_{\psi'^+} \psi'^+, Z_m m, Z_{m'} m', Z_v v, Z_{v'} v' \right\}, \\
\lambda_0 &= \lambda Z_\lambda, \quad u_0 = u Z_u, \quad u_{10} = u_1 Z_{u_1}, \quad g_{10} = g_1 \mu^\varepsilon Z_{g_1}, \\
g_{30} &= g_3 \mu^{\frac{\varepsilon}{2}} Z_{g_3}, \quad g_{40} = g_4 \mu^\delta Z_{g_4}, \quad g_{50} = g_5 \mu^{\frac{\varepsilon}{2}} Z_{g_5}.
\end{aligned} \tag{7}$$

The model is logarithmic for $\varepsilon = \delta = 0$ and the UV divergences are manifested in the form of poles in various linear combinations of ε and δ in dimensional regularization, which is very convenient for practical calculation[9, 10]. These divergences are eliminated by introducing the renormalization constants. Their explicit form depends on the choice of the subtraction scheme. Of course, universal results are independent of the choice of the particular scheme. In the minimal subtraction (MS) scheme only UV divergent terms are subtracted from the Feynman diagrams, and we use this scheme in our calculations. The facts indicated above indeed allow proving that the renormalized action has the same form as (5) and differs by the renormalized parameters and fields $Z_{\psi'^+}$, $Z_{\psi'}$, Z_{ψ^+} , Z_ψ , $Z_{m'}$, Z_m , Z_{g_1} , Z_{g_3} , Z_{g_5} , Z_u , Z_{u_1} , and Z_λ . The following relations must be satisfied for the renormalization constants of the fields in (5):

$$Z_v Z_{v'} = 1, \quad Z_m Z_{m'} = 1, \tag{8}$$

which are the consequences of the absence of the renormalization of the terms $m' \partial m$ and $v' \partial v$.

3 The UV renormalization

The RG invariance [9] can be expressed by the differential equation $D_{RG}W = 0$, where W denotes either the connected or the one-particle irreducible (1PI) Green function and the differential part of the RG operator is defined as

$$D_{RG} \equiv \mu \frac{\partial}{\partial \mu} \Big|_0 = \mu \frac{\partial}{\partial \mu} + \sum_{g_i} \beta_{g_i} \frac{\partial}{\partial g_i} - \sum_a \gamma_a a \frac{\partial}{\partial a}. \quad (9)$$

The differentiation is performed at fixed value of the bare parameters, which is indicated by the subscript "0". The first summation is over the whole set of charges $g_i = \{g_1, g_3, g_4, g_5, u, u_1\}$, and second is over the set $a = \{\lambda, h\}$. The RG functions β_{g_i} and $\gamma_F, F = a, g_i$, are given by

$$\gamma_F = \mu \frac{\partial \ln Z_F}{\partial \mu} \bigg|_0, \quad \beta_i = \mu \frac{\partial g_i}{\partial \mu} \bigg|_0. \quad (10)$$

The explicit form of the beta functions follows from this definition and relations (7). It is useful to rescale the coupling constants as


$$g_1/(8\pi^2) \rightarrow g_1, \quad g_3/\sqrt{8\pi^2} \rightarrow g_3, \quad g_4/(8\pi^2) \rightarrow g_4, \quad g_5/\sqrt{8\pi^2} \rightarrow g_5. \quad (11)$$

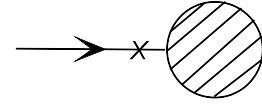
It can be seen from the perturbation expansion that the "real" coupling constants are the quadratic forms g_3^2 and g_5^2 and not simply g_3 and g_5 , whence comes the square root for g_3 and g_5 in (11). This fact is also manifested in the fixed-point coordinates because there we expect that $g_3^2 \propto \varepsilon$ and hence $g_3 \propto \sqrt{\varepsilon}$ (the same applies also for the charge g_5). Using the definitions (7) and (9), we can write β functions (10) in the forms

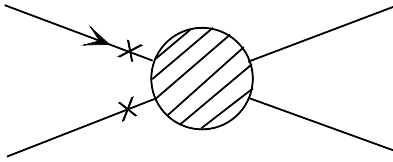
$$\begin{aligned}\beta_{g_1} &= g_1(-\varepsilon - \gamma_{g_1}), & \beta_{g_3} &= g_3(-\varepsilon/2 - \gamma_{g_3}), & \beta_u &= -u\gamma_u, \\ \beta_{g_4} &= g_4(-\delta + 3\gamma_\nu), & \beta_{g_5} &= g_5(-\varepsilon/2 - \gamma_{g_5}), & \beta_{u_1} &= -u_1\gamma_{u_1}.\end{aligned}\quad (12)$$

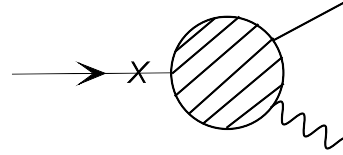
To calculate the renormalization constants in the $\overline{\text{MS}}$ scheme [9], we must subtract the UV-divergent terms (poles in ϵ and δ in our case) from the Feynman graph expansion of the corresponding 1PI functions for the given term in action (5). We can write these functions schematically in the frequency-momentum representation as


$$\Gamma_{\psi+\psi'} = 2\lambda Z_1 + \text{diagram} , \quad (13)$$

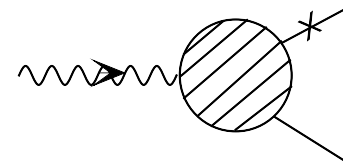
$$\Gamma_{m'm'} = 2\lambda u p^2 Z_2 + \text{diagram} \quad , \quad (14)$$


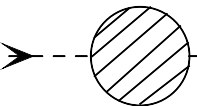
$$\Gamma_{\psi+\psi} = i\omega Z_3 - \lambda p^2 Z_4 + \text{diagram} \quad , \quad (15)$$


$$\Gamma_{\psi+\psi+\psi+\psi} = -\frac{4\lambda g_1 \mu^\varepsilon}{3} Z_5 + \text{diagram} \quad , \quad (16)$$


$$\Gamma_{\psi+\psi m} = -i\lambda g_3 \mu^{\varepsilon/2} Z_6 + \text{diagram} \quad , \quad (17)$$


$$\Gamma_{m'm} = -\lambda u p^2 Z_7 + \text{diagram} \quad , \quad (18)$$


$$\Gamma_{m'\psi+\psi} = -i\lambda g_5 \mu^{\varepsilon/2} Z_8 + \text{diagram} \quad , \quad (19)$$


$$\Gamma_{v'v} = -\nu p^2 Z_9 + \text{diagram} \quad , \quad (20)$$


where the solid non-orientable lines denote the legs formed of ψ fields. The line with an arrow denotes the response field ψ' , the lines with a cross denote the complex-conjugated

fields (i.e. ψ^+ or $\psi^{+'}$), the wavy lines denote the fields m and m' , and the dashed lines denote the field v' (line with arrow) and the field v . Shaded blobs represent all possible one-loop 1PI Feynman diagrams for the given function.

Renormalization constants (13)-(20) are related to the renormalization constants of the parameters and fields (7) via the relations

$$\begin{aligned}
Z_1 &= Z_\lambda Z_{\psi^{+'}} Z_{\psi'}, & Z_2 &= Z_\lambda Z_u Z_{m'}^2, & Z_3 &= Z_{\psi^{+'}} Z_\psi = Z_{\psi^{+'}} Z_\psi Z_v, \\
Z_3^* &= Z_{\psi'} Z_{\psi^+} = Z_{\psi'} Z_{\psi^+} Z_v, & Z_4 &= Z_{\psi^{+'}} Z_\lambda Z_\psi, & Z_4^* &= Z_{\psi'} Z_\lambda Z_{\psi^+}, \\
Z_5 &= Z_{\psi^{+'}} Z_{g_1} Z_\lambda Z_{\psi^+} Z_\psi^2, & Z_5^* &= Z_{\psi'} Z_{g_1} Z_\lambda Z_{\psi^+}^2 Z_\psi, & Z_6 &= Z_{\psi^{+'}} Z_\lambda Z_{g_3} Z_\psi Z_m, \\
Z_6^* &= Z_{\psi'} Z_\lambda Z_{g_3} Z_{\psi^+} Z_m, & Z_7 &= Z_{m'} Z_\lambda Z_u Z_m, & Z_8 &= Z_{m'} Z_\lambda Z_{g_5} Z_{\psi^+} Z_\psi, \\
Z_9 &= Z_{v'} Z_\nu Z_v.
\end{aligned} \tag{21}$$

From these relations, we can be easily obtain

$$\begin{aligned}
Z_\lambda &= Z_4 Z_3^{-1}, & Z_u &= Z_7 Z_3 Z_4^{-1}, \\
Z_{m'} &= Z_2^{1/2} Z_7^{-1/2}, & Z_m &= Z_2^{-1/2} Z_7^{1/2}, \\
Z_{g_3} &= Z_6 Z_4^{-1} Z_2^{1/2} Z_7^{-1/2}, & Z_{g_5} &= Z_8 Z_7^{1/2} Z_1 Z_4^{-2} Z_2^{-1/2} Z_3 (Z_3^*)^{-1}, \\
Z_{g_1} &= Z_5 Z_1 Z_4^{-2} (Z_3^*)^{-1}, & Z_{\psi^{+'}} &= Z_{\psi'} = Z_1^{1/2} Z_3^{1/2} Z_4^{-1/2}, \\
Z_{\psi^+} &= Z_\psi = Z_1^{-1/2} Z_3^{1/2} Z_4^{1/2}, & Z_{u_1} &= Z_9 Z_3 Z_4^{-1}, \\
Z_v &= 1, & Z_\nu &= Z_9.
\end{aligned} \tag{22}$$

We can thus obtain the anomalous dimensions γ directly from the knowledge of renor-

malization constants Z_1 - Z_9 , and in the one-loop approximation, we obtain the results

$$\begin{aligned}
\gamma_\lambda &= \frac{3g_4u_1^2}{8(1+u_1)} + \frac{g_3^2}{(1+u)^3} + \frac{g_3g_5u(2+u)}{(1+u)^3}, \quad \gamma_{g_4} = \frac{g_4}{8\delta} \\
\gamma_u &= -\frac{g_3^2}{(1+u)^3} - \frac{g_3g_5(u^3+u^2-3u-1)}{2u(1+u)^3} + \frac{3g_4u_1^2(1+u_1-uu_1-u^2)}{8u(1+u_1)(u+u_1)} \\
\gamma_{g_3} &= -\frac{3g_4u_1^2}{8(1+u_1)} - \frac{g_3^2}{(1+u)^3} + \frac{g_5^2}{4u} - \frac{g_3g_5(1+3u+11u^2+5u^3)}{4u(1+u)^3} \\
\gamma_{g_5} &= -\frac{3g_4u_1^2(1+2u+2u_1)}{8(1+u_1)(u+u_1)} - \frac{g_3g_5(5u+23u^2+9u^3-1)}{4u(1+u)^3} \\
&\quad + \frac{g_3^2(2+9u+3u^2)}{2(1+u)^3} - \frac{g_5^2}{4u} \\
\gamma_{g_1} &= -\frac{3g_4u_1^2}{4(1+u_1)} - \frac{5g_1}{3} - \frac{3g_3^2g_5(g_3-g_5)}{ug_1(1+u)} + \frac{2g_3(1+3u+u^2)(g_3-g_5)}{(1+u)^3} \\
\gamma_{u_1} &= -\frac{g_3^2}{(1+u)^3} - \frac{g_3g_5u(2+u)}{(1+u)^3} + \frac{g_4(1+u_1-3u_1^2)}{8(1+u_1)} \\
\gamma_m &= \frac{g_3g_5}{4u} - \frac{g_5^2}{4u}, \quad \gamma_{m'} = -\frac{g_3g_5}{4u} + \frac{g_5^2}{4u} \\
\gamma_\psi &= \gamma_{\psi^+} = \frac{3g_4u_1^2}{16(1+u_1)} - \frac{g_3(g_3-g_5)(2+4u+u^2)}{2(1+u)^3} \\
\gamma_{\psi'} &= \gamma_{\psi'^+} = -\frac{3g_4u_1^2}{16(1+u_1)} + \frac{g_3(g_3-g_5)u(2+u)}{2(1+u)^3}.
\end{aligned} \tag{23}$$

We note that the limit case $g_3 = g_5, g_4 = 0$ agrees with the results for model E without velocity fluctuations [3, 10].

4 Scaling regimes and fixed points' structure.

Scaling regimes are associated with fixed points of the corresponding RG functions. The fixed points are defined as such points $g^* = (g_1^*, g_3^*, g_4^*, g_5^*, u^*, u_1^*)$ at which all β functions vanish simultaneously

$$\beta_{g_1}(g^*) = \beta_{g_3}(g^*) = \beta_{g_4}(g^*) = \beta_{g_5}(g^*) = \beta_u(g^*) = \beta_{u_1}(g^*) = 0. \tag{24}$$

The type of the fixed point is determined by the eigenvalues of the matrix of its first derivatives $\Omega = \{\Omega_{ik} = \partial\beta_i/\partial g_k\}$, where β_i is the full set of β functions and g_k is the

full set of charges $\{g_1, g_3, g_4, g_5, u, u_1\}$. The IR-asymptotic behavior is governed by the IR-stable fixed points, for which all real parts of eigenvalues of matrix Ω are positive. Analysis of β functions (12) reveals, that there are several possible regimes in the case without thermal fluctuations, i.e., for $g_3 = 0$. The stable fixed points are listed in Table 1, and the unstable fixed points are listed in Table 2.

FP	FP1	FP2	FP3	FP4
g_1	0	0	$\frac{3}{5}\varepsilon$	$\frac{3}{5}\varepsilon$
g_3	0	0	$\varepsilon^{1/2}$	$\varepsilon^{1/2}$
g_5	0	0	$\varepsilon^{1/2}$	$\varepsilon^{1/2}$
g_4	0	$\frac{8\delta}{3}$	0	$\frac{8\delta}{3}$
u	0	1	1	1
u_1	0	$\frac{1+\sqrt{13}}{6}$	0	0

Table 1: Stable fixed points

FP	FP5	FP6	FP7	FP8	FP9
g_1	0	$\frac{3\varepsilon-2\delta}{5}$	$\frac{3\varepsilon-2\delta}{5}$	0	0
g_3	0	0	0	$\varepsilon^{1/2}$	$\varepsilon^{1/2}$
g_5	$\frac{\sqrt{2(-19+\sqrt{13})\delta+18\varepsilon}}{3}$	$\frac{\sqrt{2(-19+\sqrt{13})\delta+18\varepsilon}}{3}$	0	$\varepsilon^{1/2}$	$\varepsilon^{1/2}$
g_4	$\frac{8\delta}{3}$	$\frac{8\delta}{3}$	$\frac{8\delta}{3}$	0	$\frac{8\delta}{3}$
u	1	1	1	1	1
u_1	$\frac{1+\sqrt{13}}{6}$	$\frac{1+\sqrt{13}}{6}$	$\frac{1+\sqrt{13}}{6}$	0	0

Table 2: Unstable fixed points

The trivial Gaussian-like fixed point FP1 is IR-stable for $\varepsilon < 0$ and $\delta < 0$ and corresponds to the model without any nontrivial interactions. The fixed point FP2 is a

IR-stable in the region given by the inequalities $\delta > 0$ and $\delta > \frac{3}{2}\varepsilon$ and corresponds to the turbulent regime (because $g_4^* \neq 0, \delta = 4$ and $\gamma_\nu^* = \frac{\delta}{3}$).

The fixed points FP3 and FP4 differ only by the value of the charge g_4^* . The hydrodynamic fluctuations of the velocity field are IR irrelevant for FP3 and relevant, for the FP4. The fixed point FP3 is stable in the region where $\delta < 0, \varepsilon > 0$ and FP4 is stable for $\delta > 0, \delta < \frac{3}{2}\varepsilon$. Comparing FP8 and FP9 with their analogues FP3 and FP4, we can see that the absence of the interaction term $\psi^{+'}\psi^+\psi^2$ leads to system instability. We expect that this behavior can be explained by the disordering effect due to thermal fluctuations (charge $g_3 \neq 0$) because there are no other interactions between the relevant degrees of freedom (fields of the type ψ) that could stabilize system.

Briefly examining the common properties of the fixed points FP5-FP7, we see that regardless of the presence of the interaction $\psi^{+'}\psi^+\psi^2$, velocity fluctuations destabilize IR behavior.

The charges u and u_1 do not play the role of expansion parameters and it therefore seems reasonable to consider specific limits as their values tend to infinity. We consider the case where $u \rightarrow \infty$ (case I) in Table 3. To analyze this regime, we introduce new variables $w \equiv 1/u$, $f_3 \equiv g_3^2/u$, and $f_5 \equiv g_5^2/u$. Their beta functions have the form $\beta_w = w\gamma_u$, $\beta_{f_3} = f_3[-\varepsilon + \gamma_u - 2\gamma_{g_3}]$ and $\beta_{f_5} = f_5[-\varepsilon + \gamma_u - 2\gamma_{g_5}]$. The fixed points FP1^I is Gaussian (free). The fixed points FP2^I and FP3^I differ only by the value of g_4^* . The fixed point FP4^I corresponds to the turbulent regime where the interaction $\psi^{+'}\psi^+\psi^2$ is relevant. The last fixed point FP5^I is case without thermal fluctuations ($f_3 = 0$).

We consider another limit case where $u_1 \rightarrow \infty$ (case II) in Table 4. In this case, we introduce new variables $w_1 = 1/u_1$ and $f_4 = g_4 u_1$. The corresponding beta functions have the forms $\beta_{w_1} = w_1\gamma_{u_1}$ and $\beta_{f_4} = f_4[-\delta + 3\gamma_\nu - \gamma_{u_1}]$. From Table 4, we again see that the only difference between FP2^{II} and FP3^{II} is the charge g_4^* , and FP4^{II} corresponds to a turbulent regime. The fixed point FP5^{II} corresponds to a nontrivial IR-scaling regime without thermal fluctuations.

Finally, we analyze the case where both charges u and u_1 tend to infinity simultaneously (see Table 5). In the FP2^{III} regime, the presence of the interaction term $\psi^{+'}\psi^+\psi^2$ is irrelevant, unlike for the FP3^{III}. The fixed point FP4^{III} corresponds to the turbulent regime with the interaction $\psi^{+'}\psi^+\psi^2$, while that interaction is irrelevant in the regime FP5^{III}.

The last most, nontrivial case corresponds to the situation where all charges have non-zero values. But because the structure of the γ -functions is cumbersome, we have not yet found the coordinates of this fixed point and its region of stability. Of course, from other fixed points, we know where to expect such a stability region. In the near future, we hope to confirm our expectations by direct numerical calculations.

FP	FP1 ^I	FP2 ^I	FP3 ^I	FP4 ^I	FP5 ^I
g_1	0	$\frac{3\epsilon}{5}$	$\frac{3\epsilon}{5}$	$\frac{1}{5}(3\epsilon - 2\delta)$	$\frac{1}{5}(3\epsilon - 2\delta)$
f_3	0	$\frac{2\epsilon}{3}$	$\frac{2\epsilon}{3}$	0	0
f_5	0	$\frac{2\epsilon}{3}$	$\frac{2\epsilon}{3}$	0	$2\epsilon - 2\delta$
g_4	0	0	$\frac{8\delta}{3}$	$\frac{8\delta}{3}$	$\frac{8\delta}{3}$
w	0	0	0	0	0
u_1	0	0	0	$\frac{1}{6}(1 + \sqrt{13})$	$\frac{1}{6}(1 + \sqrt{13})$

Table 3: Fixed points for limiting case $u \rightarrow \infty$

FP	FP1 ^{II}	FP2 ^{II}	FP3 ^{II}	FP4 ^{II}	FP5 ^{II}
g_1	0	0	$\frac{3\epsilon}{5}$	$\frac{3}{5}(\epsilon - 2\delta)$	$\frac{3}{5}(\epsilon - 2\delta)$
g_3	0	0	0	0	0
g_5	0	$\sqrt{2\epsilon}$	$\sqrt{2\epsilon}$	0	$\sqrt{2(\epsilon - 4\delta)}$
f_4	0	0	0	$\frac{8\delta}{3}$	$\frac{8\delta}{3}$
u	0	1	1	1	1
w_1	0	0	0	0	0

Table 4: Fixed points for limiting case $u_1 \rightarrow \infty$

5 Conclusion

We have studied model E was studied in the vicinity of the critical point of the phase transition from the normal to the superfluid phase with both critical and velocity fluctuations taken into account. We showed that the model can be made multiplicatively renormalizable by adding a new charge in the interaction part of the action. We calculated the renormalization constants and RG functions up to the first order (one-loop) in the perturbation theory and partly analyzed the fixed-point structure. Our main obser-

FP	FP1 ^{III}	FP2 ^{III}	FP3 ^{III}	FP4 ^{III}	FP5 ^{III}
g_1	0	0	$\frac{3\epsilon}{5}$	$\frac{3}{5}(\epsilon - 2\delta)$	$\frac{3}{5}(\epsilon - 2\delta)$
f_3	0	$\frac{2\epsilon}{3}$	$\frac{2\epsilon}{3}$	0	0
f_5	0	$\frac{2\epsilon}{3}$	$\frac{2\epsilon}{3}$	0	$2(\epsilon - 3\delta)$
f_4	0	0	0	$\frac{8\delta}{3}$	$\frac{8\delta}{3}$
w	0	0	0	0	0
w_1	0	0	0	0	0

Table 5: Fixed points for limiting case case $u \rightarrow \infty$ and $u_1 \rightarrow \infty$

vation is that incorporation of velocity fluctuations destabilizes the critical behavior.

The work was supported by grant RFBR No.12-02-00874-a, by VEGA grant 1/0222/13 of the Ministry of Education, Science, Research and Sport of the Slovak Republic, by Center of Excellency for Nanofluid of IEP SAS and by National Scholarship Program of the Slovak Republic. This article was also created by implementation of the Cooperative phenomena and phase transitions in nanosystems with perspective utilization in nano-and biotechnology project No 26220120033, No 26110230061 and No 26220120021. Funding for the operational research and development program was provided by the European Regional Development Fund.

References

- [1] A.A. Abrikosov, L.P. Gorkov, I.E. Dzyaloshinskii, *Methods of Quantum Field Theory in Statistical Physics* (Moscow, 1998).
- [2] P.C. Hohenberg and B.I. Halperin, Rev. Mod. Phys. **49**, 435 (1977).
- [3] C. De Dominicis, L. Peliti, Phys. Rev. **B 18**, 353 (1978).
- [4] M.V. Komarova, D.M. Krasnov, M.Yu. Nalimov, Theor. Math. Phys. **169**, 89 (2011).
- [5] J. Honkonen, M.Yu. Nalimov, J. Phys. A: Gen. **22**, 751 (1989).

- [6] Honkonen J., Nalimov M.Yu., Z. Phys. B **99**, 297 (1996).
- [7] N.V. Antonov, M. Hnatic, J. Honkonen, J.Phys. A: Math. Gen. **39**, 7867 (2006).
- [8] P.C. Martin, E.D. Siggia and H.A. Rose, Phys. Rev. **A 8**, 423 (1973).
- [9] J. Zinn-Justin, Quantum Field Theory and Critical Phenomena (Clarendon Press, Oxford, 1996).
- [10] A. N. Vasil'ev, *The Field Theoretic Renormalization Group in Critical Behavior Theory and Stochastic Dynamics* (Chapman&Hall/ CRC Press, New York, 2004).
- [11] A.N. Vasil'ev, *Functional Methods in Quantum Field Theory and Statistical Physics* (Gordon and Breach Science Publishers, Amsterdam 1998).